

On break-away forces in actuated motion systems with nonlinear friction

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Abstract

The phenomenon of so-called break-away forces, as maximal actuation forces at which a sticking system begins to slide and thus passes over to a steady (macro) motion, is well known from engineering practice but still less understood in its cause-effect relationship. This note analyzes the break-away behavior of systems with nonlinear friction, which is analytically well-described by combining the Coulomb friction law with rate-independent presliding transitions and, when necessary, Stribeck effect of the velocity-weakening steady-state curve. The break-away conditions are harmonized with analytic form of the system description and shown to be in accord with a relationship between the varying break-away force and actuation force rate – well known from the experiments reported in several independently published works.

Keywords: Friction, break-away force, nonlinearity, presliding, modeling, stiction force, hysteresis

1. Introduction

The break-away force and related break-away conditions belong to significant and well-known, but still not fully studied, aspects of nonlinear dynamic friction in the actuated motion systems. The break-away as phenomenon can be seen as a brief yet non-discrete transition between the presliding and gross sliding when an idle system with friction is subject to a continuously increasing actuation (input) force. Due to the lack of measurement access and complexity of nonlinear friction transitions the break-away instant and force are particularly challenging for accurate detection and description in a closed analytic form. Some researchers even noted that "quantitative prediction of the break-away friction level seems not yet possible" [1].

The first detailed studies of presliding frictional characteristics and transitions into gross sliding may be credited to the works of Dahl, e.g. [2, 3]. Later, in the well-celebrated survey on friction modeling and control [4] the authors also addressed the break-away friction while noting that the break-away is not instantaneous and the corresponding modeling should account for translational distance. In further works on dynamic friction modeling [5, 6, 7] the authors have paid attention to, and extracted from the numerical simulations, a dependency of the break-away force on the actuation force rate. In favor to that quite similar relationships have been demonstrated in various experimental setups in [8, 6, 9]. Further accurate measurements of the presliding friction transitions, continuous sliding, static friction, and dynamic friction effects can be found in [10, 11, 12, 13].

Despite the break-away phenomenon is well known from the engineering practice and has been addressed, or at

least mentioned, is several studies on the kinetic friction, its modeling and control, less work has been dedicated to formulate the straightforward analytic conditions and derive the expressions for break-away, which would be in line with the corresponding system modeling. It seems that an explicit analysis and math notation of break-away states has been solely provided in [14], while the break-away force has been rather considered as a function of dwell time, and the given deviations seem less suitable for a direct practical use.

With this note we address the relationship between the break-away friction force and actuation force rate in a possibly simple and, at the same time, coherent way based on the established modeling assumptions and results published in several independent works. The following analysis and presentation should contribute to better understanding the frictional break-away behavior and help in predicting and controlling the actuated presliding transitions.

2. Sliding and presliding friction

The tangential friction force, acting in opposite direction to the relative motion in x coordinates, is the generalized nonlinear function

$$F = f(\dot{x}, z, t). \quad (1)$$

The velocity argument can be seen as capturing the steady-state friction behavior including the amplitude-constant and $\text{sign}(\dot{x})$ -dependent Coulomb friction, the viscous velocity-dependent friction, and Stribeck velocity-weakening curves as well. All three can be described by the well-known steady-state characteristic curve

$$F_{ss}(\dot{x}) = \text{sign}(\dot{x}) \left(F_c + (F_s - F_c) \exp(-|\dot{x}|^\delta V^{-\delta}) \right) + \delta \dot{x}, \quad (2)$$

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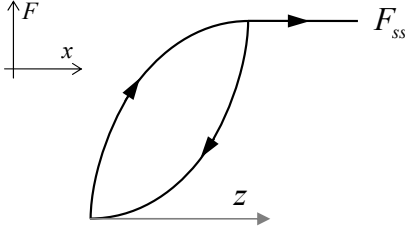


Figure 1: Friction-displacement curve with hysteresis loop.

often referred as static Stribeck friction model. The free parameters are the Coulomb friction coefficient $F_c > 0$, Stribeck (or stiction) friction level $F_s > F_c$, linear viscous friction coefficient $\delta \geq 0$, and two shape factors of the velocity-weakening curve $V > 0$ and $\delta \neq 0$. For more details on the Stribeck effect and steady-state characteristic friction curve (2) we refer to [15, 4].

The time-dependency of friction (1) summarizes the weakly known and often non-deterministic fluctuations in the frictional behavior due to e.g. wear, adhesion effects, contact surface irregularities, lubrication conditions, dust and others. Such effects may cause some non-systematic parameters drifting of friction modeling and, in what follows, are excluded from an explicit consideration.

The z -argument represents an internal presliding state or so-called relative presliding distance on the frictional interface. Most simple way, this is a relative displacement at each motion onset or motion reversal until the dynamic friction force converges to the steady-state of gross sliding at an unidirectional motion. Obviously, the presliding distance z is initialized (or reset) whenever the velocity sign changes and maps explicitly the instantaneous state of presliding friction force transitions. Depending on the particular form of presliding friction $F = f(z)$, as a function of relative presliding distance, an additional scaling factor s may be used so that

$$z = s \int_{t_r}^t \dot{x} dt. \quad (3)$$

Obviously, t_r denotes the time instant of the last motion reversal so that $|z|$ represents always a scaled distance to the position where the motion direction changed for the last time. For the sake of simplicity we will assume in the following $s = 1$. The friction-displacement curve of a contact surface at motion reversals exhibit a hysteresis loop, see Figure 1. The shape of hysteresis loops depends on multiple factors of the asperities interaction, their elastic and plastic deformation and, as a consequence, on energy dissipated on the frictional surface during the motion cycles [16, 17]. According to [18] the area of hysteresis loop increases in proportion to the n -th power of presliding distance. In particular, it has been found and experimentally proved that this is the second power, i.e. $n = 2$. Therefore, the curvature of friction-displacement map during preslid-

ing is given by

$$f(z) = z(1 - \ln(z)). \quad (4)$$

For details on deriving of equation (4) from the above n -th, respectably second, power condition we refer to [18].

Assuming the hysteresis loop shape (4) and $s = 1$ it is obvious that for zero initial state $F_0 = 0$ at the motion onset the presliding friction is given by

$$F(\dot{x}, z) = \text{sign}(\dot{x}) F_{ss}(\dot{x}) z(1 - \ln(z)). \quad (5)$$

Assuming the presliding transitions always converge to the steady-state F_{ss} and the instantaneous friction value at the last motion reversal is F_r the friction force in presliding is given by

$$F(\dot{x}, z) = |\text{sign}(\dot{x}) F_{ss}(\dot{x}) - F_r| z(1 - \ln(z)) + F_r. \quad (6)$$

Note that the normalized, through the scaling factor s , presliding distance is defined on the interval $[-1, 1]$, while at the boundaries the friction force converges to the steady-state value.

3. Break-away conditions

The problem of break-away friction force can be seen as a problem of detection (alternatively prediction) of the minimal actuation force at which the motion system, being initially in the idle state, begins the continuous (macro) motion, often denoted as gross sliding. This problem is closely related to the stiction and adhesion effects on the complex frictional interfaces and, at the same time, is of empirical observation nature and relevance in the engineering practice. The transitions from the system sticking to gross sliding at an unidirectional motion have been observed in various actuated machines and mechanisms and reported in e.g. [10, 4, 12]. In most the previously published works the varying break-away force (or torque) has been exposed in dependency of the actuation (input) force rate. That means the actuation (input) force has been linearly increased starting from zero, i.e. $u = kt$, and the break-away transition has been observed and recorded as when a non-fluctuating quasi-constant acceleration occurs and the relative velocity grows continuously. Note that before this, the system is in presliding regime where a low relative displacement can be detected, while the measured relative velocity is mostly high-frequent oscillating around zero, with a relatively low average and its increase, see e.g. experiments depicted in Fig. 8 in [19]. The dependency of the observed break-away force on the actuation force rate $du/dt = k$ has been investigated and experimentally demonstrated in [8, 6, 9], and also shown for the numerically simulated dynamic friction in [5, 6, 7]. In all cases a typical inverse exponential map has been highlighted as schematically shown in Figure 2 (cf. e.g. Fig. 4 in [5], Fig. 5 in [6], Fig. 13 in [7], Fig. 10 in [9]).

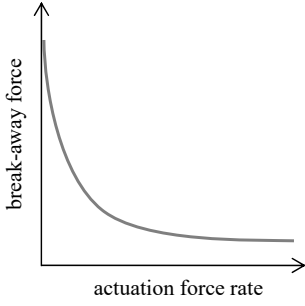


Figure 2: Break-away force as function of the actuation force rate.

Now, we are in the position to analyze and describe analytically the break-away conditions based on the modeling assumptions made in Section 2. We note that despite the break-away dependency on the actuation force rate has been known from experiments and confirmed by means of numerical simulations, no explicit analytic form has been derived and validated so far in line with the modeled presliding friction behavior.

For motion dynamics with a linearly increased actuation force we write

$$m\ddot{x} + f(\dot{x}, z) = kt \quad (7)$$

During presliding, the macroscopic system inertia can be neglected due to a very low acceleration so that the actuation force is mainly balanced by the counteracting friction, so that $f(\dot{x}, z) \approx kt$. Taking the time derivative of (7) and neglecting the inertial dynamics one obtains

$$\frac{d}{dt}f(\dot{x}, z) = k, \quad (8)$$

while the full differential yields [20]

$$\frac{d}{dt}f(\dot{x}, z) = \frac{\partial f}{\partial \dot{x}}\ddot{x} + \frac{\partial f}{\partial z}\dot{z}. \quad (9)$$

For the same reason as above and due to the fact that $\partial f / \partial \dot{x} = 0$ within presliding, the first right-hand-side summand in (9) can be neglected and we obtain

$$\frac{\partial f}{\partial z}\dot{z} - k = 0. \quad (10)$$

Substituting the derivative of (4), with respect to z , into (10) results in

$$-F_{ss} \ln(z) \dot{z} = k. \quad (11)$$

It is obvious that the relative velocity during presliding

$$\dot{z} = -\frac{k}{F_{ss} \ln(z)} \quad (12)$$

can be computed as a function of relative presliding distance and depends mainly on two factors k and F_{ss} . While k is fixed for the given slope of external actuation force, the steady-state friction value self depends on the instantaneous relative velocity. Nevertheless, from (2) we know

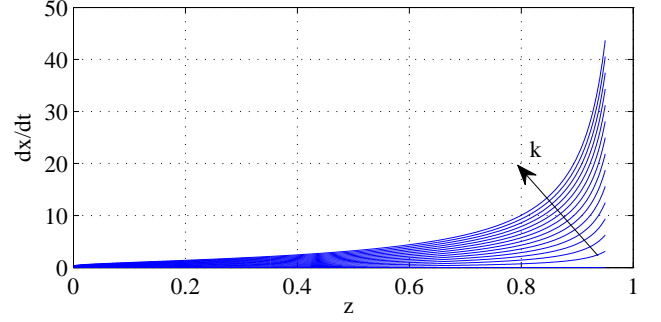


Figure 3: Relative velocity as a function of presliding distance in dependency of the rate of actuation force.

that $F_c \leq |F_{ss}| \leq F_s$ so that either both boundary values or an average

$$\hat{F}_{ss} = F_c + (F_s - F_c)/2$$

can be assumed for calculating (12).

The (z, \dot{x}) phase diagrams are shown in Figure 3 for the different actuation force rates k and a fixed \hat{F}_{ss} value. One can see that for all actuation force rates the relative velocity, starting from zero after a motion reversal ($z = 0$), increases exponentially when approaching boundary of the presliding range ($z = 1$). The computed phase diagrams are for the actuation force rates $k \in [0.01, \dots, 30]$ with an increment equal 2. Since a rapid (exponential) increase of the relative velocity is for all k when $z \rightarrow 1$ one can restrict the considered values by e.g. 95 % of presliding distance, denoted by $z_{0.95}$. Here we should note that transition from the presliding to the gross sliding is not abrupt/stepwise at all, and the break-away conditions can be considered only for a certain, though well-specified interval, like for example $0.95 < z < 1$ we assumed. This is also in accord with the experimental and numerical observations reported so far, while the break-away detection is mostly realized “at the time where a sharp increase in the velocity could be observed” [5].

For the assumed presliding boundary the break-away force can be computed, based on (2) and (12), as

$$F_{ba} = F_{ss}(\dot{x}(z_{0.95})). \quad (13)$$

Now one can calculate the break-away force as a function of actuation force rate k and that provided the friction model (2)-(4) only is given. The assumed steady-state (Stribeck) characteristic curve is depicted in Figure 4. Note that the linear viscous friction coefficient $\sigma = 0$ is assumed for the sake of simplicity, and a relatively high difference $F_s = 1.5F_c$ between the minimal and maximal steady-state friction values is chosen. The computed break-away force as a function of actuation force rate is depicted in Figure 5. In order to reveal the impact of F_{ss} , assumed for (12) computation, the minimal (F_c), maximal (F_s), and averaged (\hat{F}_{ss}) steady-state values are demonstrated opposite to each other. One can see that the functional

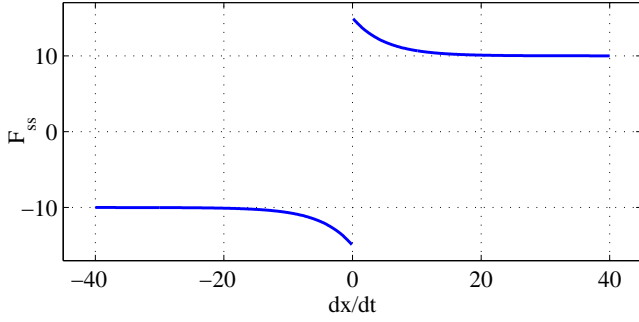


Figure 4: Steady-state (Stribeck) characteristic friction curve.

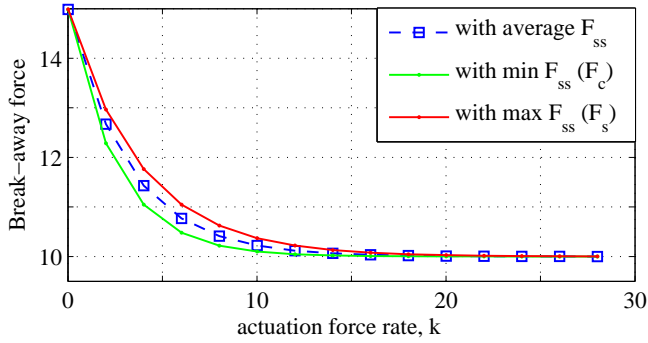


Figure 5: Break-away force as function of the actuation force rate computed by (2), (12), and (13) for minimal (F_c), maximal (F_s), and average (\bar{F}_{ss}) steady-state friction values.

dependency of break-away force from k is similar for all three steady-state values. In particular, at lower (near to zero) and higher actuation force rates they nearly coincide with each other. On the opposite, the differences mostly occur at the well pronounced exponential decrease.

4. Conclusions

This notice aimed to analyze and describe analytically the break-away conditions for which, at certain level of the external actuation force and its rate, the presliding friction behavior transits to the gross sliding and a continuous (macro) motion sets on. Using the straightforward formulation of the presliding and steady-state friction force it has been explicitly shown how the relative velocity progresses with the relative presliding distance starting from zero idle state, and when a rapid exponential increase of the relative velocity which is characteristic for break-away occurs. We have derived an analytic expression for computing the break-away force as a function of actuation force rate. The computed and exposed results are in accord with the previously published experimental observations and those from the numerical simulations for which, however, an analytic expression and analysis have been missed.

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